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SYNTHESIS OF FEEDBACK SYSTEMS WITH NONLINEAR UNCERTAIN PLANTS,(U)
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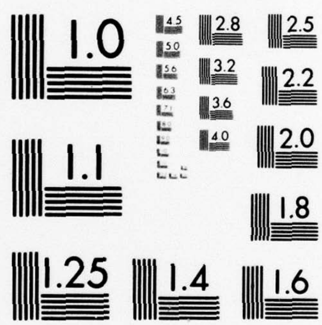

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (19) (18) AFOSR-TR- 78-1570	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) SYNTHESIS OF FEEDBACK SYSTEMS WITH NONLINEAR UNCERTAIN PLANTS	(2) 5C	5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Baxter F. Womack ~~and Hin~~ Wo Lo /Hin		8. CONTRACT OR GRANT NUMBER(s) (15) F49620-77-C-0101
9. PERFORMING ORGANIZATION NAME AND ADDRESS Electronics Research Center, University of Texas at Austin, Austin, Texas 78712		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2305/A9 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS AF Office of Scientific Research/NE, Building #410, Bolling AFB, D.C. 20332		12. REPORT DATE (11) March 1978
		13. NUMBER OF PAGES 5 pages
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (16) 2305 (17) A9		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

(12) 7 p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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18. SUPPLEMENTARY NOTES

Proceedings of 1978 IEEE Region V Annual Conference

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

nonlinear synthesis design; uncertain plant equivalence; time domain tolerance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A synthesis theory for feedback systems with nonlinear uncertain plants to satisfy prescribed output tolerances is presented. For any element in the finite input set,

$I = \{i^\alpha, \alpha = 1,2,\ldots,N\}$, there is specified a set of acceptable responses, $Z^\alpha(\omega)$. The theory guarantees that for any $i^\alpha \varepsilon I$, the system responses satisfy the corresponding $Z^\alpha(\omega)$ over the range of plant uncertainty. The essence of the theory is to convert

the nonlinear plant set to a linear time-invariant one for which a design procedure exists. Schauder's fixed point theorem is applied to prove the equivalence of these two plant sets.

AFOSR-TR- 78-1570

SYNTHESIS OF FEEDBACK SYSTEMS WITH NONLINEAR UNCERTAIN PLANTS

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Abstract

A synthesis theory for feedback systems with nonlinear uncertain plants to satisfy prescribed output tolerances is presented. For any element in the finite input set, $I=\{i^\alpha,\alpha=1,2,\ldots,N\}$, there is specified a set of acceptable responses, $Z^\alpha(\omega)$. The theory guarantees that for any $i^\alpha \epsilon I$, the system responses satisfy the corresponding $Z^\alpha(\omega)$ over the range of plant uncertainty. The essence of the theory is to convert the nonlinear plant set to a linear time-invariant one for which a design procedure exists. Schauder's fixed point theorem is applied to prove the equivalence of these two plant sets.

1. Problem Description

A nonlinear plant is imbedded in a single-input, single-output, two-degree-of-freedom feedback structure as shown in Fig. 1. The nonlinear plant is assumed to be time invariant. Because of parameter uncertainty, the plant is given as a denumerable set of mappings, $W=\{w_i\}$, one for each different parameter combination. The system input is given as a finite set $R=\{r(s)\}$. All extraneous signals are lumped as a set of disturbance inputs d at the output of the system. Let $I=\{i^\alpha\}$ denote the set of inputs for which the design is to be executed. Each element of I may consist of a command input, a disturbance input, or a combination of both. For each $i^\alpha \epsilon I$, there is specified a distinct set of acceptable plant ouputs, denoted as $Z^\alpha(\omega)=\{z_\kappa^\alpha(\omega)\}$. (The subscript κ is usually dropped unless it is needed explicitly.) The objective is to choose compensating networks F and G such that, for any $i^\alpha \epsilon I$, the output is guaranteed to satisfy the performance specifications.

As might be noticed, the specification is given in the frequency domain. This is because the synthesis theory presented in this paper is based on the availability of a synthesis technique for linear time-invariant (LTI) systems, which exists for frequency domain specification; no such design procedure is available for the time domain specification. However, the specification can be given in the time domain provided it can be translated to the frequency domain. While no exact translation exists, there is little difficulty in approximating one for engineering purposes. Two methods are currently available: (1) The approximation can be done on a digital computer using the Fast Fourier Transform algorithm. However, the algorithm is intended for periodic waveforms; when applying to nonperiodic waveforms, the error introduced can be quite large, especially at high frequency, due to the aliasing effect. The error can be reduced to some extent by choosing the sampling period sufficiently small. (2) The approximation can also be done by assuming a simple second- or third-order system model in frequency domain, and finding its bounds on the model parameters which correspond to the bounds on the time response.

2. Synopsis

As noted earlier, the synthesis theory for nonlinear feedback systems presented here is based on the availability of a design technique for LTI system. Hence, in order to apply this LTI design technique, it is necessary to convert the nonlinear plant set to a linear one. The derivation of this LTI plant set from the nonlinear plant set and the set of acceptable outputs is presented in Section 3.

Section 4 gives a brief account of the LTI technique for the design of feedback systems with linear uncertain plant. Application of this technique to feedback systems with nonlinear uncertain plant will also be discussed.

Replacing the nonlinear plant set by the LTI plant set and applying the LTI design technique to the system shown in Fig. 1 yields two compensation blocks-F and G. The remaining problem is to show that when the nonlinear plant replaces the LTI plant, the output also satisfies the performance criteria. This is done in Section 5. In this section, Schauder's fixed point theorem is applied to prove the equivalent of the nonlinear plant set and the LTI plant set derived in Section 3. This chapter in essence proves the validity of the synthesis theory.

A design example which illustrates the synthesis theory is presented in Section 6.

3. Derivation of LTI Plant Set

3.1 Generation of the Set of Acceptable Output Time Functions

For any $z^\alpha(\omega)$, $\ln z^\alpha(\omega)$ may be considered as one of a pair of conjugate functions, specifically as the real part of

$$\ln z^\alpha(j\omega)=\ln|z^\alpha(j\omega)|+j \arg z^\alpha(j\omega).$$

Using Hilbert Transform,

$$\arg z^\alpha(j\omega)=\frac{2\omega}{\pi}\int_0^\infty \frac{\ln z^\alpha(\beta)}{\beta^2-\omega^2}\,d\beta$$

then

$$z^\alpha(j\omega)=z^\alpha(\omega)\ \mathrm{Exp}\{j \arg z^\alpha(j\omega)\}$$

and

$$z^\alpha(s)=z^\alpha(j\omega)\big|_{s=j\omega}.$$

To assure that $\arg z^\alpha(j\omega)$ and $z^\alpha(s)$ are unique, $z^\alpha(s)$ must be a minimum phase function (1), i.e. it has no finite zeros in the right-half of the complex plane. Using the inverse Laplace Transform, $z^\alpha(t)$ can be obtained from $z^\alpha(s)$.

In view of the foregoing discussion, each $z^\alpha(j\omega)$ uniquely generates, via $z^\alpha(j\omega)$ and $z^\alpha(s)$, a $z^\alpha(t)$. Hence, a set of output time functions $Z^\alpha(t)=\{z^\alpha(t)\}$ is obtained from the set of acceptable responses $Z^\alpha(\omega)$. This set of time functions will be used to generate the LTI plant set.

3.2 Generation of the LTI Plant Set

An important step in the synthesis theory is the generation of a LTI plant set, one for each $i^\alpha \epsilon I$, from the nonlinear plant set and the set of acceptable responses (for that α value).

Consider any pair of $z^\alpha(t)\epsilon Z^\alpha(t)$ and $w_i \epsilon W$, the input to the plant is given by

$$x_{\kappa i}^\alpha(t)=(w_i)^{-1}\, z_\kappa^\alpha(t)$$

provided the inverse exists. To assure this, a

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restriction is imposed on the nonlinear plant set.

(R3): The nonlinear time invariant plant is a denumerable set of mappings $w:x(t)\to z(t)$ with unique continuous inverses whose values are Laplace transformable.

Using the Laplace Transform, define

$$w^{\alpha}_{\kappa i}(s)=\frac{z^{\alpha}_{\kappa}(s)}{x^{\alpha}_{\kappa i}(s)}.$$

$w^{\alpha}_{\kappa i}(s)$ is the LTI transfer function which is equivalent to w_i in the sense that, for a specific system input i^{α}, the input $x^{\alpha}_{\kappa i}(s)$ applied to $w^{\alpha}_{\kappa i}(s)$ will give the same output as the input $x^{\alpha}_{\kappa i}(s)$ applied to w_i.

Repeating the above procedure for all $w_i \varepsilon W$ gives the LTI plant set $w^{\alpha}_{\kappa}(s)=\{w^{\alpha}_{\kappa i}(s)\}$. $w^{\alpha}_{\kappa}(s)$ is equivalent to W in the sense that when i^{α} is fixed, the set of plant inputs required to obtain $z^{\alpha}_{\kappa}(s)$ as $w^{\alpha}_{\kappa i}(s)$ ranges over $w^{\alpha}_{\kappa}(s)$ is precisely the same as when the nonlinear plant element w_i ranges over W.

Repeating the procedure again for all $z^{\alpha}(s)\varepsilon Z^{\alpha}(\omega)$ gives $W^{\alpha}(s)=\{w^{\alpha}(s)\}$. This LTI plant is equivalent to W for any pair of $z^{\alpha}_{\kappa}(s)\varepsilon Z^{\alpha}(s)$ and $w_i \varepsilon W$. This equivalence is good only for a specific system input i^{α}. In general, different sets are obtained for different system inputs.

Once the LTI plant set is obtained, the LTI design procedure can be applied. To assure the applicability of this design procedure, a restriction is imposed on $W^{\alpha}(s)$.

(R4): All $w^{\alpha}(s)\varepsilon W^{\alpha}(s)$ are minimum phase.

This restriction is imposed because only for such plants does the linear design technique guarantee any tolerances are achievable regardless of how large the plant uncertainty may be, providing an additional restriction (which will be discussed in Section 4) is also satisfied.

4. Synthesis of Uncertain Plants

4.1 LTI Technique

In Section 3, the nonlinear plant set was converted to the LTI plant set. The next logical step is to carry out the LTI design procedure for this LTI plant set. However, it seems appropriate to give a brief introduction of this LTI design technique first[1,2]. This section will discuss the LTI design technique as well as the application of this technique to feedback systems with nonlinear uncertain plants.

Consider the feedback structure depicted in Fig. 2. P denotes the transfer function of a LTI uncertain plant. The plant parameters are not known precisely, only the ranges of their values are known. These plant parameters, though uncertain, are assumed to be fixed. (Strictly speaking, this LTI design technique is applicable only to a fixed parameter plant; however, it is also applicable to slow-varying parameters for engineering purposes.) F and G are the transfer functions of two compensation networks to be designed such that the system output satisfies the prescribed tolerances; an example of each tolerances is depicted in Fig. 3.

4.2 Design Procedure

1) Translation of $|C(j\omega)|$ to $|T(j\omega)|$ which is the magnitude of the system transfer function. If the input is chosen to be a unit impulse, then $|T(j\omega)|$ is the same as $|C(j\omega)|$.

2) Derivation of bounds on $L(j\omega)\triangleq G(j\omega)P(j\omega)$.

3) Shaping of $L(j\omega)$.

4) Derivation of $G(j\omega)$ and $F(j\omega)$.

The effect of feedback is to reduce the sensitivity of system response with respect to plant parameter variation. The measure of this sensitivity is given by the sensitivity function $S^T_P=1/(1+L)$. The idea is to choose $L(j\omega)$ such that the system exhibits low sensitivity over some frequency range of interest. This is achieved if $S^T_P<1$. Obviously, $S^T_P\to 0$ as $|L(j\omega)|\to\infty$, thus $L(j\omega)$ should be chosen as large as possible. However, any practical $L(j\omega)$ must go to zero at infinite ω, so it is necessary that $L(j\omega)$ be allowed to go to zero. To assure that this can be done, the following restriction is invoked:

$$\text{(R5):}\ \lim_{\omega\to\infty}\frac{\ln\{b(\omega)/a(\omega)\}}{\Delta\ \ln\ |P(j\omega)|}>1.$$

Restriction (R5) simply means that, at sufficiently high frequencies, the plant parameter variation is smaller than the permitted closed-loop output variation. Consequently, feedback is not necessary since sensitivity reduction is, of course, not needed.

The satisfaction of Restriction (R5), then, guarantees that there exists ω_h such that $\omega>\omega_h$, $B(\omega)$ lies inside a finite closed region Γ in the complex plane, enclosing the (-1,j0) point but not the origin[6]. The existence of Γ assures the existence of an acceptable $L(j\omega)$. The fact that Γ does not enclose the origin is crucial, because it is the dynamic ω-equivalent of the critical (-1,j0) point in the complex plane, and it must be suitably bypassed as $L(j\omega)\to 0$ at infinite ω. If Γ does not enclose the origin, then $L(j\omega)$ will be able to find its way to the origin without crossing Γ.

In view of the foregoing discussion, $L(j\omega)$ can be shaped in the following way. For $\omega<\omega_h$, $L(j\omega)$ should be sufficiently large; for $\omega>\omega_h$, $L(j\omega)$ should decrease for a certain interval to avoid Γ, and then go to zero at the rate required by its excess of poles over zeros. $L(j\omega)$ can be shaped this way if it is a minimum phase function.

In the discussion of the LTI design procedure, only one fixed input and the corresponding set of acceptable responses is considered. The design produces a single set of boundary curves, $B(\omega)$. The design procedure can be extended to a set of N inputs (say step functions of different magnitudes), and one set of acceptable outputs for each of the N inputs. Because of the superposition principle, an input of 2u(t) will produce an output twice as large as the unit step response. Consequently, there is only one transfer function for the system, and the LTI design will still produce only one single set of $B(\omega)$. However, this is not the case in nonlinear feedback systems simply because a unique transfer function just does not exist. As a result, a different set of $B^{\alpha}(\omega)$ is obtained for each $i^{\alpha}\varepsilon I$. To apply the LTI design, a single $B(\omega)$ is found, at each ω, which satisfies the requirements for all $B^{\alpha}(\omega)$. Once this set of $B(\omega)$ is obtained, the design procedures can proceed as discussed in previous sections.

5. Equivalence of $W^{\alpha}(S)$ and W

By applying the LTI design technique to the LTI plant set derived in Section 3, there emerges with two compensation networks F and G. The design guarantees that: if $i^{\alpha}\varepsilon I$ is the system input, then the output satisfies $Z^{\alpha}(\omega)$ regardless of which $w^{\alpha}(s)\varepsilon W^{\alpha}(s)$ happens to represent the plant. It remains to be proven, however, that the output performance is also satisfactory

when the nonlinear plant set W replaces the LTI plant set $W^\alpha(s)$. Suppose a hypothetical output sequence, paired with $w_i \varepsilon W$, generates the LTI plant set. Then, this plant set is equivalent to W if and only if the actual output sequence is precisely the same as the hypothetical sequence used to derive the LTI plant set. In this section, Schauder's fixed point theorem is applied to show that the nonlinear plant set W and the LTI plant set $W^\alpha(s)$ are equivalent.

5.1 Schauder's Fixed Point Theorem

Schauder's fixed point theorem states that "a continuous operator mapping a closed convex compact set of a Banach space into itself has a fixed point." The space of continuous real functions $C[0,\infty)$ with $||z^\alpha(\omega)|| = \underset{\omega}{\mathrm{Sup}}|z^\alpha(\omega)|$ is used as the Banach space. An additional restriction is imposed on $Z^\alpha(\omega)$ to assure that it a closed, convex, and compact set in the Banach space.

(R6): $\frac{d\, z^\alpha(\omega)}{d\omega}$ is uniformly bounded, i.e. there exists K>0 such that $|\frac{d\, z^\alpha(\omega)}{d\omega}| \leq K$ for all ω, and for all $z^\alpha(\omega)\varepsilon Z^\alpha(\omega)$.

It follows from Schauder's theorem that there exists at least one fixed point $\bar{z}_0^\alpha(\omega)$ such that $\phi^\alpha(\bar{z}_0(\omega)) = \bar{z}_0(\omega)$.

5.2 Application of Schauder's Theorem

Having shown that $\phi^\alpha : \bar{z}^\alpha(\omega) \to \bar{q}^\alpha(\omega)$ is a continuous operator mapping a closed, convex, and compact set into itself, and that in accordance with Schauder's fixed point theorem, a fixed point exists such that $\phi^\alpha(\bar{z}_0^\alpha(\omega)) = \bar{z}_0^\alpha(\omega)$, we are now in a position to show that the LTI plant set $\bar{W}^\alpha$ and the nonlinear plant set W are equivalent.

At the outset of the formulation of the synthesis theory, a hypothetical output sequence $\bar{z}^\alpha(\omega)$, paired with the nonlinear plant sequence $\bar{w}$, generates the LTI plant sequence $\bar{w}^\alpha(s)$. Using this plant sequence, the LTI design emerges with two compensating networks F and G such that when i^α is applied to the system, and the components of $\bar{w}^\alpha(s)$ are inserted into the system one by one, then the output sequence of the system is $\bar{q}^\alpha(\omega)$. The LTI design guarantees that $\bar{q}^\alpha(\omega)\varepsilon \bar{Z}^\alpha(\omega)$. However, it does not guarantee that $\bar{q}^\alpha(\omega)$ is the same sequence which was used to generate $\bar{q}^\alpha(\omega)$. In practice, $\bar{W}^\alpha(s)$ is equivalent to W if and only if the actual output sequence $\bar{q}^\alpha(\omega)$ is precisely the same as the hypothetical output sequence used to generate $\bar{w}^\alpha(s)$. Although the LTI design does not guarantee this, it has been proven that a fixed point $\bar{z}_0(\omega)$ does exist such that $\phi^\alpha(\bar{z}_0^\alpha(\omega)) = \bar{z}_0^\alpha(\omega)$. This particular output sequence generates $\bar{w}_0^\alpha(s)$ which, through the design of F and G, gives back $\bar{z}_0^\alpha(\omega)$. Hence the LTI plant sequence may be replaced by the nonlinear plant sequence $\bar{w}$, the resulting plant input sequence as well as the output sequence will be exactly the same in either case, i.e. the two systems are indistinguisable.

6. Design Example

6.1 Introduction

In this section, a design example is given to illustrate the synthesis theory. The set of acceptable responses is represented by the frequency-domain tolerances shown in Fig. 4, the corresponding time-domain tolerances are shown in Fig. 5. The nonlinear plant is described by the equation $x(t) = kz^2(t)$, where $k \varepsilon [0.5,2]$. The input set consists of a unit step. The objective is to design two compensation networks F and G (Fig. 1) such that the system's responses to the unit step satisfy the prescribed tolerances over the range of plant uncertainty.

6.2 Synthesis Procedure

Three output curves and six different values of k are chosen. Following the procedure discussed in Section 3, an 18-point transfer function is generated. The LTI design technique discussed in Section 4 is then employed to derive a set of bounds on $L(j\omega)$. One possible set of such bounds for selected values of ω is shown in Fig. 6; also shown in the figure is the chosen $L(j\omega)$. The series compensation network is

$$G(s) = \frac{3.5(s+1.5)(s+100)}{s+0.2)(s+3)(s+16)} .$$

It turns out that, for this particular case, $|L/1+L|$ satisfies the prescribed tolerances, hence no prefilter is needed.

6.3 Discussion of Results

Simulation results are shown in Fig. 7. The curves labeled 1 through 6 correspond to six different values of k ranging from 0.5 to 2 at a uniform increment of 0.3, respectively. The results are considered satisfactory.

It should be noted that the specifications are satisfied only for those inputs belonging to the input set. For any other inputs, it is conceivable that the system responses may not be satisfactory or even unbounded due to the nonlinear nature of the system. The responses of the system subjected to an input of 5u(t) is depicted in Fig. 8, which can be seen as rather sluggish. In practice, if any input not in the input set gives undesirable performances, then it can be added to the input set and the whole system redesigned.

References

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Acknowledgements

This research was supported in part by the DoD Joint Services Electronics Program through the Air Force Office of Scientific Research (AFSC) Contract F49620-77-C-0101.

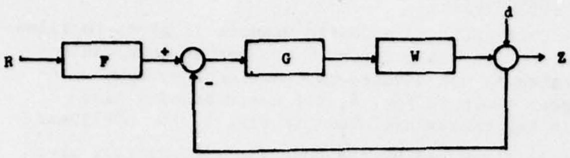

Figure 1. Feedback system with nonlinear uncertain plant.

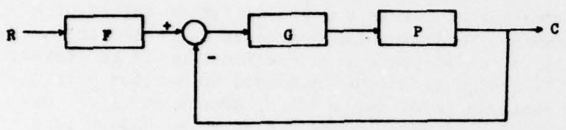

Figure 2. Feedback system with LTI uncertain plant.

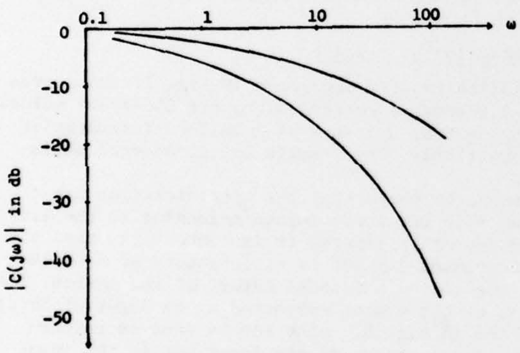

Figure 3. Bounds on |C(jω)|.

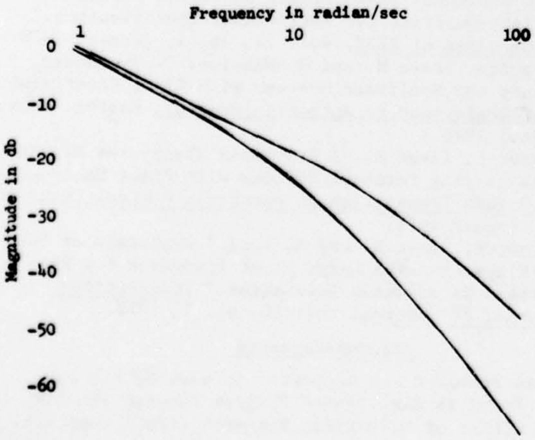

Figure 4. Frequency-domain tolerances.

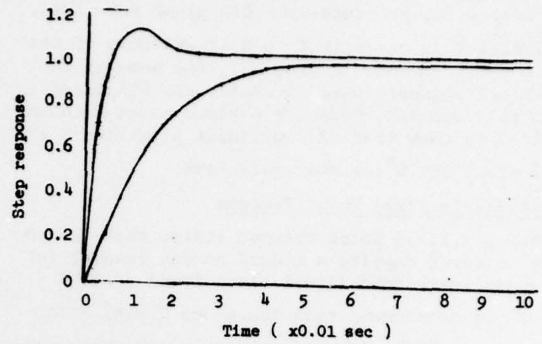

Figure 5. Time-domain tolerances.

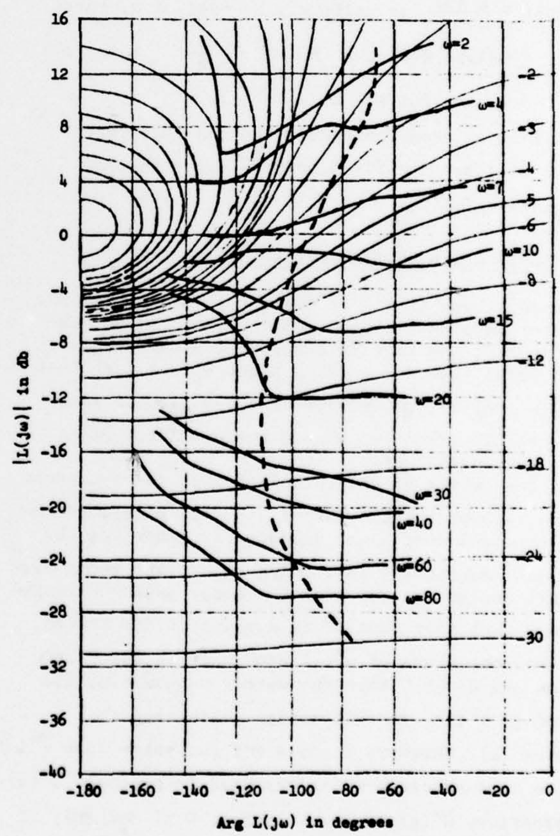

Figure 6. Bounds on L(jω) and the chosen L(jω).

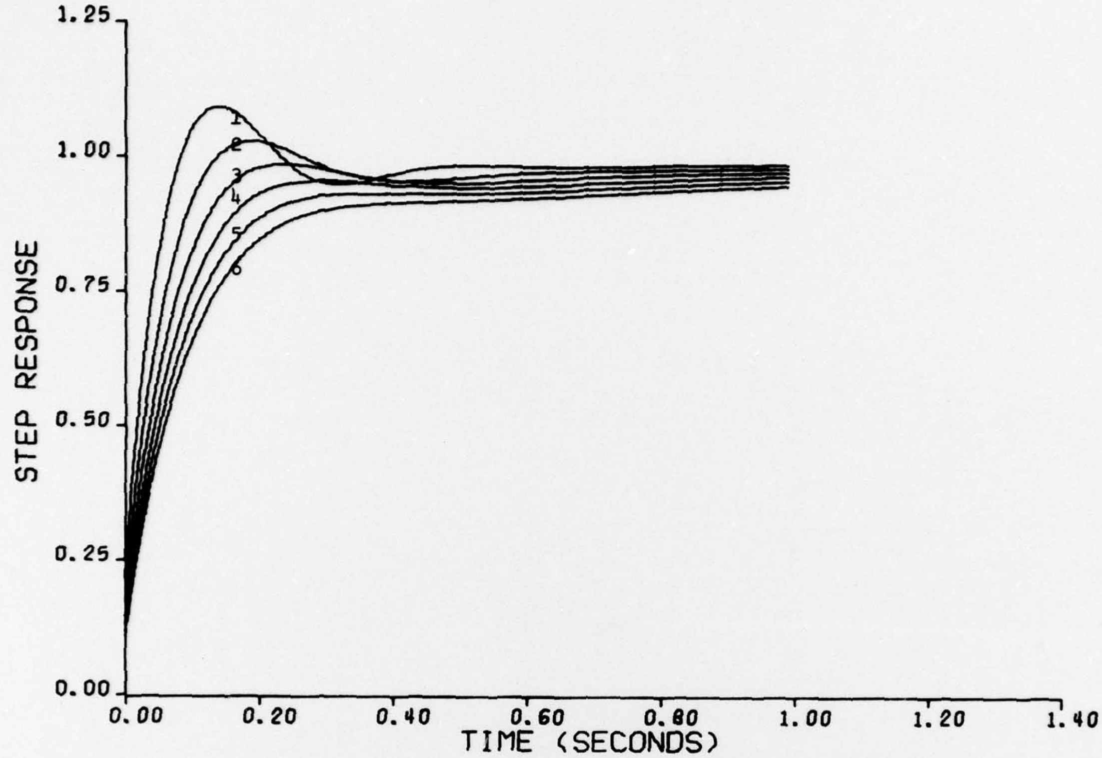

Figure 7. System response to an input of u(t).

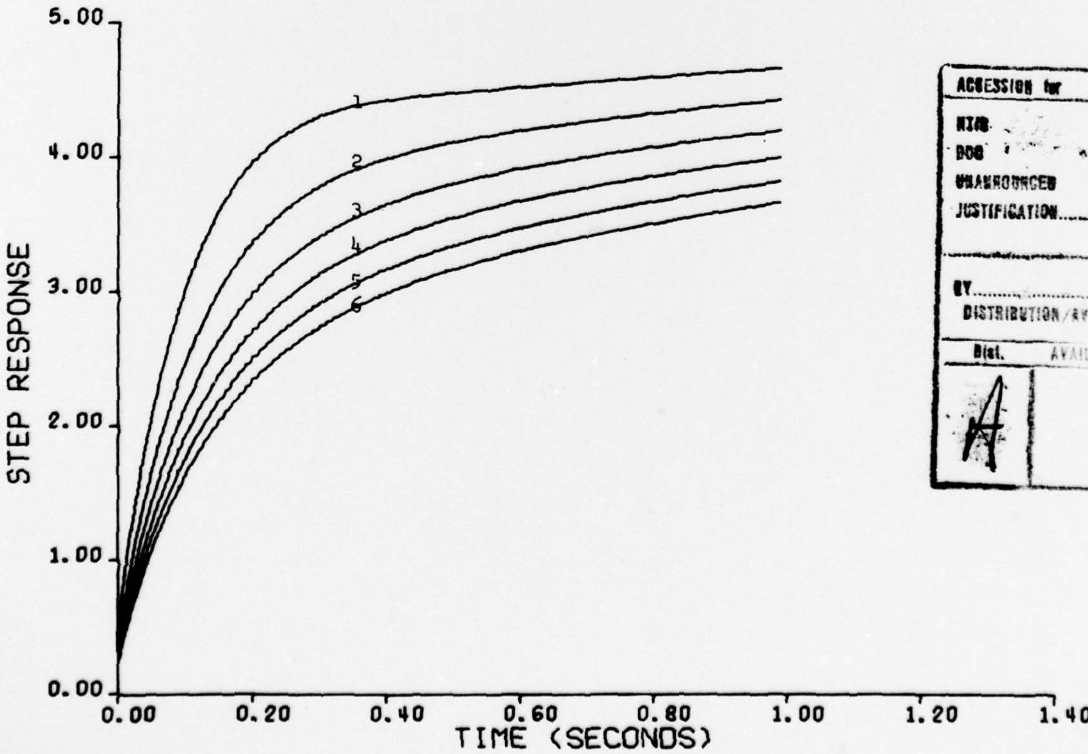

Figure 8. System responses to an input of 5u(t).